

$$A) \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!} =$$

$$15) \sum_{n=1}^{\infty} (n+1)! x^n =$$

Find the interval of Convergence and the function that represents the series

$$24) \sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{9^n} =$$

$$28) \sum_{n=0}^{\infty} \left(\frac{\sin x}{2} \right)^n =$$

CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watts and Kennedy
 Chapter 9: Convergence of Series

What you'll Learn About
 Testing Endpoints for Convergence

RADIUS OF CONVERGENCE

$$x = -\frac{1}{2}$$

$$\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n \left(-\frac{3}{2}\right)^n$$

$$\sum_{n=0}^{\infty} (-1)^n$$

$$\lim_{n \rightarrow \infty} (-1)^n \neq 0$$

Find I.O.C.

Geometric Ratio Test

$$x = 0$$

$$\sum \frac{(-1)^n (-1)^n}{n^2}$$

$$\sum \frac{1}{n^2} p=2 > 1$$

converges

24) $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n (x-1)^n =$

$$r = \frac{2}{3}(x-1)$$

$$x = \frac{5}{2}$$

$$\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n \left(\frac{3}{2}\right)^n$$

$\sum (1)^n$ diverges
 $\lim_{n \rightarrow \infty} 1 \neq 0$

5) $\sum_{n=0}^{\infty} \frac{(-1)^n (3x-1)^n}{n^2} =$

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} (3x-1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n (3x-1)^n} \right| = |3x-1| < 1$$

$$x = \frac{2}{3} \rightarrow \text{Abs. Convergence}$$

$$\sum \frac{(-1)^n}{n^2}$$

the terms decrease in abs value to 0
 and $\sum \frac{1}{n^2}$ converges

$$-1 < \frac{2}{3}(x-1) < 1$$

$$-3 < 2(x-1) < 3$$

$$-\frac{3}{2} < x-1 < \frac{3}{2}$$

$$-\frac{1}{2} < x < \frac{5}{2} \quad \text{I.O.C.}$$

$$\text{R.D.C.} = \frac{3}{2}$$

$$\frac{(3x-1)^n (3x-1)}{(n+1)^2} \cdot \frac{n^2}{(3x-1)^n} = \frac{n^2 (3x-1)}{(n+1)^2}$$

$$-1 < 3x-1 < 1$$

$$0 < 3x < 2$$

$$0 \leq x \leq \frac{2}{3}$$

$$a) -1 \leq x \leq 1$$

b) None

$$c) x = -1$$

$$9) \sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}} =$$

$$\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{x^n} \right| = |x| < 1$$

$$-1 \leq x < 1$$

Conditional convergence
b/c the terms
dec in abs. value to 0

$$x = 1 \\ \sum \frac{(-1)^n}{\sqrt{n}}$$

$$x = 1 \\ \sum \frac{1}{\sqrt{n}}, p = \frac{1}{2} \leq 1 \text{ diverges}$$

$$13) \sum_{n=0}^{\infty} \frac{n!}{2^n} x^{2n} =$$

$$\frac{(n+1)x^2}{2}$$

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1)!}{2^{n+1}} x^{2n+2} \cdot \frac{2^n}{x^{2n} n!} \right| = \infty > 1$$

Diverges, except
at the center $x=0$

$$1) \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} =$$

$$\lim_{n \rightarrow \infty} \left| \frac{1}{(n+1)} \cdot (-x) \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{(-x)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-x)^n} \right| = |x| < 1$$

$$|x| < 1$$

I.O.C.: $-1 < x < 1$

$$-\infty < x < \infty$$

2015 AP Calculus BC Free Response

6. The Maclaurin Series for a function f is given by

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \dots + \frac{(-3)^{n-1}}{n} x^n + \dots$$

and converges to $f(x)$ for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

a) Use the Ratio Test to find R .

20. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converges?

- (A) $-3 \leq x \leq 3$ (B) $-3 < x < 3$ (C) $-1 < x \leq 5$ (D) $-1 \leq x \leq 5$ (E) $-1 \leq x < 5$